NASA TECHNICAL MEMORANDUM NASA TM X-64577



THREE INVESTIGATIONS OF THE INTERSTELLAR MEDIUM

By Klaus Schocken Space Sciences Laboratory Science and Engineering

February 11, 1971

NASA

George C. Marshall Space Flight Center Marshall Space Flight Center, Alabama

т	E	CH	INI	CAL	_ RE	PORT	r st/	\N E)AR	D TIT	1 E	P	٩G	E

	TECHNI	CAL REPORT STANDARD TITLE PAGE
1. REPORT NO. NASA TM X-64577	2. GOVERNMENT ACCESSION NO.	3. RECIPIENT'S CATALOG NO.
4. TITLE AND SUBTITLE	<u> </u>	5. REPORT DATE
Three Investigations of the Interstel	lar Medium	February 11, 1971 6. PERFORMING ORGANIZATION CODE
7. AUTHOR (S) Klaus Schocken	<u> </u>	8. PERFORMING ORGANIZATION REPORT #
9. PERFORMING ORGANIZATION NAME AND A	DDRESS	10. WORK UNIT NO.
J. TEM SHIMING SHOWING THE PROPERTY OF THE PRO		
George C. Marshall Space Flight Cer Marshall Space Flight Center, Alaba		11. CONTRACT OR GRANT NO.
12. SPONSORING AGENCY NAME AND ADDRES	30	13. TYPE OF REPORT & PERIOD COVERED
National Aeronautics and Space Ad		Technical Memorandum
Washington, D. C. 20546		14. SPONSORING AGENCY CODE
15. SUPPLEMENTARY NOTES	and the second of the second 	en in maligne de la primitation de la companie de
Prepared by Space Sciences Laborat	cory, Science and Engineering	
is negligible. Two types of clouds he hydrogen clouds carry a small admit composition of nonvolatile element by the equation of radiative transfers. It is concluded that an attent and oxygen in the cosmic proportion tied up not in clouds in space but in attenuating medium, a cloud in space graphite spheres surrounded with me are accompanied by 10 molecules of the intergalactic space, the definition of the clouds in the cosmic proportion in the cosmic proportion attenuating medium, a cloud in space graphite spheres surrounded with me are accompanied by 10 molecules of the cosmic proportion in the cosmic proportion	r for an absorbing-scattering medium nuating cloud consists of the element ons; the heavier elements neon, magn a stellar plasmas and stellar clouds. A ce consists of a hydrogen plus helium nantles of ices. The nitrogen is tied to	hydrogen and clouds of dust. The st clouds has essentially the cosmic through clouds has to be determined in. Its hydrogen, helium, carbon, nitrogen, tesium, silicon, sulfur, and argon are according to this model of the in mixture in which are embedded up in ammonia; 100 molecules of H ₂ O in comparison with the interstellar
17. KEY WORDS		ied-Unlimited A Challen
19. SECURITY CLASSIF. (of this report)	20. SECURITY CLASSIF, (of this page)	21. NO. OF PAGES 22. PRICE
Unclassified	Unclassified	30 \$3.00

The gas can be considered to be hydrogen with the cosmic fraction of helium. The other components form the dust particles. The particles possess a core of graphite on which are adsorbed H_2 O and NH_3 . The interstellar gas is not evenly distributed but is present as clouds. The average size of a cloud is of the order of $3.0856 \cdot 10^{17}$ m. The magnetic field and the cosmic ray gas drive the interstellar gas-field system unstable in periods of about $(3.156 \cdot 10^7)^7$ sec. As a result of the instabilities, turbulence develops. In a plasma confined by a magnetic field, whose value remains constant in time at each point, the diffusion velocity across the magnetic field, resulting from collisions of electrons with ions, can be obtained. Under the assumptions made in Section III, the turbulent diffusion rate is of the same order of magnitude as the collisional diffusion rate.

TABLE OF CONTENTS

	Page
SUMMARY	1
I. THE ATTENUATION OF X-RAYS IN SPACE	1
A. The Absorption and Scattering Coefficients B. The Distribution of Interstellar Matter C. Radiative Transfer in an Absorbing-Scattering Cloud	7
II. THE ATTENUATION OF X-RAYS IN THE INTERSTELLAR MEDIUM AND IN INTERGALACTIC SPACE	13
A. The Interstellar Medium B. The Attenuation of X-Rays C. Intergalactic Space	14
III. THE PHYSICAL STATE OF AN H I CLOUD	17
A. Introduction B. The Interstellar Medium C. The H I Clouds D. The Stability of H I Clouds	18 19
REFERENCES	23

LIST OF ILLUSTRATIONS

Figure	Title	Page
1.	Bihemispherical reflectance and transmittance versus albedo for different incident intensity distribution; $\tau = 0.1 \dots$	12
2.	Bihemispherical reflectance and transmittance versus albedo for different incident intensity distribution; $\tau = 0.5 \dots$	12
3.	Bihemispherical reflectance and transmittance versus albedo for different incident intensity distribution; $\tau = 1.0 \dots$	12
	LIST OF TABLES	
Table	Title	Page
1.	Attenuation and Scattering Coefficients and Albedo for Scatter for X-Rays in Space	4
2.	Optical Depth and Loss of Magnitude for a Distance $x = 3.4818 \cdot 10^{17}$ m in a Medium of Cosmic Composition in the Narrow-Beam Approximation	6
3.	Mean Particle Density of Neutral and Ionized Hydrogen at Different Distances from the Galactic Center	7
4.	Relative Distribution of Hydrogen Perpendicular to the Galactic Plane	7
5.	Parameters of Standard Cloud	8
6.	Transmittance I/I _O for Isotropic Multiple Scatter in a Slab	13
7.	Approximate Characteristic Values of the Interstellar Medium	13
8.	Attenuation Coefficients for X-Rays of a Mixture of Five Elements Between Hydrogen and Oxygen	15
9.	Transmittance I/I_O of X-Rays for a Distance $x = 3.4818 \cdot 10^{17}$ m in a Medium of Varying Cosmic Composition in the Narrow-Beam Approximation	16
10.	Relative Abundances of Some of the Light Elements	18

TECHNICAL MEMORANDUM X-64577

THREE INVESTIGATIONS OF THE INTERSTELLAR MEDIUM

SUMMARY

The attenuation of X-rays in space is concentrated in clouds. Outside the clouds, the attenuation is negligible. Two types of clouds have to be distinguished: clouds of hydrogen and clouds of dust. The hydrogen clouds carry a small admixture of helium. The dust in the dust clouds has essentially the cosmic composition of nonvolatile elements. The transmission of X-radiation through clouds has to be determined by the equation of radiative transfer for an absorbing-scattering medium.

It is concluded that an attenuating cloud consists of the elements hydrogen, helium, carbon, nitrogen, and oxygen in the cosmic proportions; the heavier elements neon, magnesium, silicon, sulfur, and argon are tied up not in clouds in space but in stellar plasmas and stellar clouds. According to this model of the attenuating medium, a cloud in space consists of a hydrogen plus helium mixture in which are embedded graphite spheres surrounded with mantles of ices. The nitrogen is tied up in ammonia; 100 molecules of H_2 O are accompanied by 10 molecules of NH_3 .

In intergalactic space, the density of grain material is negligible in comparison with the interstellar space, but the Doppler effect in expanding spaces may become significant for X-ray galaxies.

The gas can be considered to be hydrogen with the cosmic fraction of helium. The other components form the dust particles. The particles possess a core of graphite on which are adsorbed H_2 O and NH_3 . The interstellar gas is not evenly distributed but is present as clouds. The average size of a cloud is of the order of $3.0856 \cdot 10^{17}$ m. The magnetic field and the cosmic ray gas drive the interstellar gas-field system unstable in periods of about $(3.156 \cdot 10^7)^7$ sec. As a result of the instabilities, turbulence develops. In a plasma confined by a magnetic field whose value remains constant in time at each point, the diffusion velocity across the magnetic field, resulting from collisions of electrons with ions, can be obtained. Under the assumptions made in Section III, the turbulent diffusion rate is of the same order of magnitude as the collisional diffusion rate.

I. THE ATTENUATION OF X-RAYS IN SPACE

A. The Absorption and Scattering Coefficients

The attenuation of a narrow X-ray beam is measured in terms of an attenuation coefficient μ

$$I = I_0 e^{-\mu x}$$

$$\mu = \alpha + \sigma$$
.

The term I_O denotes the incident intensity and I the intensity after traversing the thickness x. The attenuation consists of a loss due to photoelectric absorption α , and a loss due to scattering σ . The terms μ , α , and σ have the dimension $[\ell^{-1}]$. The attenuation coefficient is usually tabulated in terms of

$$\kappa = \frac{\mu}{\rho}$$

since the opacity κ is more independent of the temperature than μ . The density is denoted by ρ , and κ has the dimension $[\ell^2 m^{-1}]$. The dimensionless optical thickness τ is connected with the other attenuation parameters by the relations

$$d\tau = \mu dx = \kappa \rho dx$$

The atomic attenuation coefficient k is defined by the relation

$$k = \frac{\mu}{\rho} \frac{A}{N_0}$$

$$k = k_{a} + k_{\sigma} ,$$

where A denotes the atomic weight and N_{o} is Avogadro's number:

$$N_0 = 6.022169 \times 10^{23}$$

In terms of k, which has the dimension $[\ell^2]$, the attenuation coefficient of a mixture or compound is

$$\mu = \sum_{\mathbf{Z}} \mathbf{k}_{\mathbf{Z}} \mathbf{n}_{\mathbf{Z}} ,$$

where Z denotes the atomic number and n the number of atoms per unit volume.

For high energies, Compton scattering dominates. For longer wavelengths and high atomic numbers, scattering becomes small compared to the total attenuation.

The mass scattering coefficient depends only on the number of electrons per unit weight in the absorber and is otherwise independent of the material. The scattering per electron σ_e is given by the Klein-Nishina equation. The mass scattering coefficient is given by the expression:

$$\frac{\sigma}{\rho} = \sigma_e N_o \frac{Z}{A}$$
, $k_\sigma = \sigma_e Z$

Since Z is a pure number, σ_e has the same dimension as k, $[\ell^2]$.

The cross sections of the elements k_{α} and σ_{e} were obtained by computer calculation from Nagasaka's formula and from the Klein-Nishina law, respectively, as given by Grodstein [1]. The term k_{σ} was obtained from σ_{e} , then k from k_{α} and k_{σ} , and finally μ according to the formulas stated previously. For the cosmic abundances, Aller's values [2] were used. The same substances were entered into the computations with the same abundances as given by Bell and Kingston [3]. For hydrogen, a concentration of 10 atoms cm⁻³ was assumed, and ω was obtained as the quotient.

$$\omega = \frac{\sigma}{\mu}$$

Table 1 contains the attenuation and scattering coefficients and the albedo for scatter ω , computed as described, for hydrogen, for hydrogen and the cosmic fraction of helium, and for the cosmic composition of ten elements between hydrogen and argon.

Table 2 contains the optical depth and the loss of magnitude, which is proportional to it, for a distance which should approximate a standard cloud of radius $2.1599 \cdot 10^{17}$ m in the narrow-beam approximation. The term τ is converted into astronomical magnitudes by multiplication with

$$2.5 \log e = 1.08573$$

TABLE 1. ATTENUATION AND SCATTERING COEFFICIENTS AND ALBEDO FOR SCATTER FOR X-RAYS IN SPACE

(10-10 m)		Hydrogen		Hydı	Hydrogen + Helium	elium	Cosm	Cosmic Composition	ition
(m or)	η	Q	3	π	Q	3	Ħ	Ø	3
		10-23			10-23			10-23	
0.01	0.1896	0.1896	1.0	0.2511	0.2511	1.0	0.2543	0.2543	1.0
0.02	0.2637	0.2637	1.0	0.3492	0.3492	1.0	0.3536	0.3536	1.0
0.03	0.3126	0.3126	1.0	0.4140	0.4140	1.0	0.4192	0.4192	1.0
0.04	0.3492	0.3492	1.0	0.4625	0.4625	1.0	0.4683	0.4683	1.0
0.05	0.3783	0.3783	1.0	0.5010	0.5010	1.0	0.5073	0.5073	1.0
90:0	0.4022	0.4022	1.0	0.5327	0.5327	1.0	0.5394	0.5393	1.0
0.07	0.4223	0.4223	1.0	0.5593	0.5593	1.0	0.5663	0.5663	1.0
80.0	0.4395	0.4395	1.0	0.5820	0.5820	1.0	0.5894	0.5893	0.9998
60.0	0.4543	0.4543	1.0	0.6017	0.6017	1.0	0.6094	0.6092	0.9997
0.10	0.4673	0.4673	1.0	0.6189	0.6189	1.0	0.6268	0.6266	0.9997
0.20	0.5426	0.5426	1.0	0.7187	0.7186	0.9999	0.7299	0.7276	0.9968
0.30	0.5764	0.5764	1.0	0.7638	0.7633	0.9993	0.7827	0.7729	0.9875
0.40	0.5957	0.5956	0.9998	0.7899	0.7887	0.9985	0.8265	0.7986	0.9641
0.50	0.6083	0.6079	0.9993	0.8077	0.8051	0.9968	0.8786	0.8152	0.9278
09:0	0.6174	0.6166	0.9987	0.8214	0.8166	0.9942	0.9521	0.8268	0.8684
0.70	0.6243	0.6229	0.9978	0.8333	0.8250	0.9900	1.0603	0.8353	0.7878
0.80	0.6300	0.6278	0.9965	0.8448	0.8315	0.9843	1.2188	0.8419	0.6908
06.0	0.6350	0.6317	0.9948	0.8567	0.8367	0.9767	1.4465	0.8471	0.5856
1.0	0.6396	0.6349	0.9927	0.8697	0.8408	0.9668	1.7672	0.8513	0.4817
2.0	0.7031	0.6497	0.9241	1.1887	0.8605	0.7239	26.39	0.8713	0.0330

TABLE 1. (Concluded)

Wavelenoth		Hydrogen		Hydr	Hydrogon ± Holium	Ti.i.	1		14:03
MAYOUSELL		11) di Ogoli		Liyur	Ogen - Tre	illulli	COSIIIK	COSITIC COINDOSITION	TIOII
(10-10 m)	η	O	3	η	ď	3	Ħ	ь	3
		10^{-23}			10^{-23}			10^{-23}	
3.0	0.8758	0.6550	0.7479	2.2319	0.8674	0.3886	46 856.	0.8783	0
4.0	1.2628	0.6578	0.5209	4.6297	0.8712	0.1882	9278.9	0.8821	
5.0	1.9820	0.6595	0.3327	9.1348	0.8735	0.0956	437 564.	0.8844	
0.9	3.1671	0.6609	0.2087	16.62	0.8754	0.0527		0.8863	
7.0	4.9690	0.6654	0.1339	28.07	0.8813	0.0314		0.8923	
8.0	7.5414			44.56					
0.6	11.06			67.31					
10.	15.72			97.60					
20.	173.2			1176.					
30.	723.5			5236.					
40.	2006.			136 863.	.,				
								,	

TABLE 2. OPTICAL DEPTH AND LOSS OF MAGNITUDE FOR A DISTANCE $x=3.4818\cdot 10^{17}~m$ IN A MEDIUM OF COSMIC COMPOSITION IN THE NARROW-BEAM APPROXIMATION

Wavelength	Optical Depth	Loss of Magnitude
(10 ⁻¹⁰ m)	10-4	
0.01	0.8854	0.9613
0.02	1.2312	1.3367
0.03	1.4596	1.5847
0.04	1.6305	1.7703
0.05	1.7663	1.9177
0.06	1.8781	2.0391
0.07	1.9717	2.1408
0.08	2.0522	2.2281
0.09	2.1218	2.3037
0.10	2.1824	2.3695
0.20	2.5414	2.7592
0.30	2.7252	2.9588
0.40	2.8777	3.1244
0.50	3.0591	3.3214
0.60	3.3150	3.5992
0.70	3.6918	4.0082
0.80	4.2436	4.6074
0.90	5.0364	5.4682
1.0	6.1530	6.6805
2.0	91.88	99.76
3.0	163 143.0	177 129.0
4.0	32 307.0	35 077.0
5.0	1 523 510.0	1 654 121.0

B. The Distribution of Interstellar Matter

The interstellar medium is concentrated in the galactic plane and is arranged in clouds along spiral arms. Usually gas and dust appear together. In the galactic plane, the mean density of H I is $1.4 \cdot 10^{-21}$ kg m⁻³, that of dust is $4.3 \cdot 10^{-24}$ kg m⁻³. Above the galactic plane, concentrations of coulds are less important than they are in the galactic plane. The density of gas in the halo is about 10^{-24} kg m⁻³.

The intergalactic attenuation of radiation, although small, is not negligible. The intergalactic plasma has a density of about 10 protons m⁻³.

Tables 3 and 4 show the spatial distribution of interstellar hydrogen [4], where N_{H} is the mean particle density of neutral hydrogen and $N_{H\ II}/N_{H\ I}$ represents the density ratio of ionized to neutral hydrogen.

The actual space distribution of interstellar material is highly irregular. In the simplest model picturing these irregularities, spherical dust clouds of the same radius 1, are dispersed through space with parameters about equal to those in Table 5 [5].

TABLE 3. MEAN PARTICLE DENSITY OF NEUTRAL AND IONIZED HYDROGEN AT DIFFERENT DISTANCES FROM THE GALACTIC CENTER

TABLE 4. RELATIVE DISTRIBUTION OF HYDROGEN PERPENDICULAR TO THE GALACTIC PLANE

r (3.09 · 10 ¹⁹ /m)	N _H (10 ⁶ m ⁻³)	N _{H II}
0-2 2-3 3-3.5 3.5-4 4-4.5 4.5-5 5-6	0.33 0.33 0.72 0.72 0.68	0 0.05 0.18 0.32 0.10 0.06 0.05
6-7 7-8 8-9 9-10 10-11 11-12 12-13 13-14 14-15	0.97 0.72 0.48 0.41 0.55 0.40 0.21 0.09 0.04	0.03 0.02 0.03 0.04 0

z' (3.09 · 10 ¹⁶ m)	$\frac{N_{\rm H}(2')}{N_{\rm H}(0)}$
0	1.00
20	0.98
40	0.91
60	0.81
80	0.69
100	0.56
120	0.44
140	0.33
160	0.26
180	0.21
200	0.18
250	0.12
300	0.08
350	0.06
400	0.04
450	0.02
500	0.01

TABLE 5. PARAMETERS OF STANDARD CLOUD

Radius of dust cloud	2.16 · 10 ⁷ m
Number per 2.9378 × 10 ⁵⁸ m ³ , f	5 × 10 ⁴
Number in line of sight per 3.0856×10^{19} m, $\pi 1^2$ f	8
Fraction of volume occupied, $4 \pi 1^3 f \times 1/3$	0.07
Photographic extinction in single cloud	0.02 mag
Mass	400 M _O
Density of neutral hydrogen	10 ⁷ m ⁻³

The value of the mass of a standard cloud is computed on the assumption that the ratio of dust to hydrogen by mass has the average value of 1 percent. For more detailed comparisons with observations, the model must be made more complicated.

A large cloud has a mass of $1.8 \cdot 10^4 \text{ M}_{\odot}$, a radius of $6.17 \cdot 10^{17} \text{ m}$, and a density of $n = 2.10^7 \text{ H m}^{-3}$. The largest stable cloud is about 1300 M_{\odot} . The mass of an unstable cloud, where star formation is occurring, is about $4 \cdot 10^4 \text{ M}_{\odot}$ with $n_H = 2 \cdot 10^7 \text{ m}^{-3}$.

The development of a cloud is controlled by collisions with other clouds. The time interval between collisions is about $(3.155 \cdot 10^7)^7$ sec.

A small cloud starts its life near an expanding H II region of high density compressed by surrounding hot gas. Its self-gravitation is negligible. It is accelerated outward by the expanding H II region and undergoes inelastic collisions with other clouds in the vicinity, which produce new clouds of larger mass by coalescence. Through repeated coalescence processes, the mass of the cloud increases to a degree that self-gravitation becomes significant and ultimately dominant. At this point, the cloud becomes gravitationally unstable, undergoes rapid collapse, and becomes subject to gravitational fragmentation. The final result is a group of young stars, the brightest of which ionize gas that remains in the cloud to form H II regions. These regenerate small clouds which are available for a new cycle of activity.

C. Radiative Transfer in an Absorbing-Scattering Cloud

An approximate method to obtain intensities throughout an absorbing isotropically scattering unidimensional system as a function of both position and angle has been described by Hottel, Sarofim, and Sze [6]. For a medium which itself is not an emitter, the transport equation is

$$\gamma \frac{\mathrm{d}\mathrm{I}(\tau,\gamma)}{\mathrm{d}\tau} = \mathrm{I}(\tau,\gamma) - \frac{\omega}{2} \int_{-1}^{1} \mathrm{I}(\tau,\gamma') \,\mathrm{d}\gamma' \tag{1}$$

The term on the left-hand side of equation (1) represents the total change in intensity over a distance dl. This change equals the difference between the rates of decrease in intensity due to absorption and scatter and the rate of increase due to the scatter into the direction of propagation of the incident beam by radiation incident from all directions. The γ represents the cosine of the angle made by the incident beam with the direction of increasing τ . An approximate method for solving equation (1) is the representation of the integral by a polynomial in τ . Then equation (1) reduces to a first-order linear nonhomogeneous differential equation as follows:

$$-\gamma \frac{\mathrm{d}I(\tau,\gamma)}{\mathrm{d}\tau} = I(\tau,\gamma) - \sum_{i=0}^{n-1} A_i \tau^i$$
 (2)

The formal solution of equation (2) is

$$I(\tau,\gamma) = e^{-\tau/\gamma} \int \frac{1}{\gamma} e^{\tau/\gamma} \sum_{i=0}^{n-1} A_i \tau^i d\tau + C e^{-\tau/\gamma}$$

$$I(\tau,\gamma) = \sum_{i=0}^{n-1} \left[A_i(\tau^i - i\gamma\tau^{i-1} + i(i-1)\gamma^2 \tau^{i-2} \right]$$

$$-\ldots + (-1)^{i} i! \gamma^{i}$$
 + C $e^{-\tau/\gamma}$

The integration constant is denoted by C and is evaluated from the boundary conditions. The boundary condition applicable in the present case is perfectly diffuse radiation incident on one surface of the cloud.

$$I_{+}(0,\gamma) = 1$$

$$I_{-}(\tau_1,\gamma) = 0$$

The following values of C result from these conditions:

$$C_{+} = 1 - \sum_{i=0}^{n-1} A_{i}(-1)^{i} i! \gamma^{i}$$

$$C_{-} = \left[-\sum_{i=0}^{n-1} A_{i}(\tau_{1}^{i} - i\gamma \tau_{1}^{i-1} \tau \ldots) \right] e^{\tau_{1}/\gamma}$$

After the values of C are determined, the values of A_i have to be obtained. Comparing equations (1) and (2) shows

$$\frac{\omega}{2} \int I(\tau, \gamma') d\gamma' = \sum_{i=0}^{n-1} A_i \tau^i$$

$$\frac{\omega}{2} \int_{-1}^{0} I_{\underline{}}(\tau, \gamma') d\gamma' + \frac{\omega}{2} \int_{0}^{1} I_{\underline{}}(\tau, \gamma') d\gamma' = \sum_{i=0}^{n-1} A_{i} \tau^{i}$$
 (3)

When the values of C_+ and C_- are substituted into equation (3) and the integrals are evaluated, the following equations result:

$$\begin{split} \sum_{i=1}^{n-1} \ A_i \, \tau^i &= \omega \sum_{i=0}^{n-1} \ A_i \left[\tau^i + \frac{i(i-1)}{3} \, \tau^{i-2} + \frac{i(i-1)(i-2)(i-3)}{5} \, \tau^{i-4} + \ldots \right] \\ &- \frac{\omega}{2} \, \sum_{i=0}^{n-1} \ A_i (-1)^i \, i! E_{i+2}(\tau) \\ &+ \frac{\omega}{2} \, \sum_{i=0}^{n-1} \ A_i \left[\tau^i_1 \, E_2 (\tau_1 - \tau) + i \tau^{i-1}_1 \, E_3 (\tau_1 - \tau) \right. \\ &+ i (i-1) \, \tau^{i-2}_1 \, E_4 (\tau_1 - \tau) \, \tau \ldots \right] + \frac{\omega}{2} \, \int_0^1 \, e^{-\tau/\gamma} \, d\gamma \\ E_j(\tau) &= \int_0^1 \, \gamma^{j-2} \, e^{-\tau/\gamma} \, d\gamma \end{split}$$

An exact solution to equation (1) requires that equation (3) be satisfied for all values of τ . This would require an infinite number of terms in the polynomial. An adequate accuracy can be obtained by using an $(n-1)^{th}$ -degree polynomial and satisfying equation (3) exactly at n values of τ . The coefficients A_i are determined by the solution of the n first-order linear simultaneous equations obtained when the right and left sides of equation (3) are equated at n values of τ .

The integrated reflectance and transmittance for hemispherical incidence of isotropic radiation is presented in Figures 1 through 3 [6] as a function of ω for $\tau_1 = 0.1$ to 1.0. At the depth τ , the local integral intensity is

$$\int_{-1}^{1} I(\tau, \gamma') d\gamma'$$

The influence of multiple scattering is shown in Table 6. It is seen that the errors can be considerable if multiple scattering is disregarded.

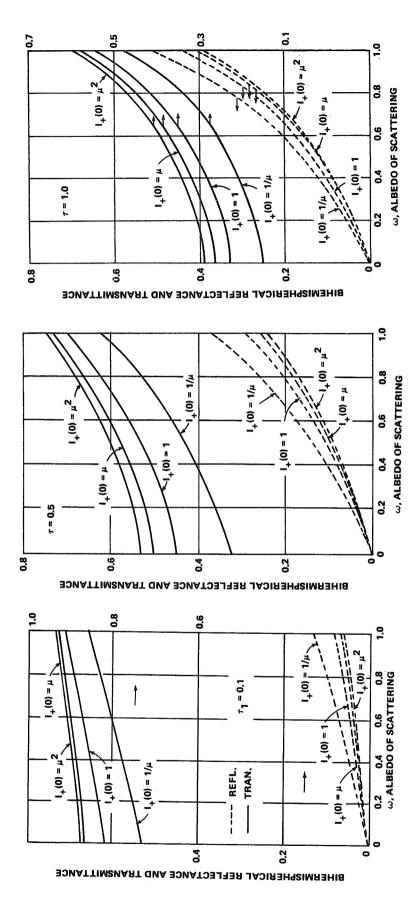


Figure 1. Bihemispherical reflectance and transmittance versus albedo for different incident intensity distribution; $\tau = 0.1$. Figure 2. Bihemispherical reflectance versus and transmittance versus albedo for different incident intensity distributions; $\tau = 0.5$.

Figure 3. Bihemispherical reflectance and transmittance versus albedo for different incident intensity distributions; $\tau = 1.0$.

TABLE 6. TRANSMITTANCE I/I_O FOR ISOTROPIC MULTIPLE SCATTER IN A SLAB

	0.1	0.5	1
Narrow-beam approximation	0.9048	0.6065	0.3679
$ \left\{ \begin{array}{l} \text{Multiple} \\ \text{scattering} \end{array} \right. \left.\begin{array}{l} \omega = 0 \\ \omega = 1 \end{array} $	0.60 0.70	0.45 0.70	0.33 0.65

II. THE ATTENUATION OF X-RAYS IN THE INTERSTELLAR MEDIUM AND IN INTERGALACTIC SPACE

A. The Interstellar Medium

The approximate characteristic values of the interstellar medium are given in Table 7.

TABLE 7. APPROXIMATE CHARACTERISTIC VALUES OF THE INTERSTELLAR MEDIUM

Extent	9.2568 · 10 ²⁰ m
Mean density, ρ	1.2 · 10 ⁻²¹ kg m ⁻³
Particle density of hydrogen in clouds	10 ⁷ m ⁻³
Temperature	10² - 10⁴ °K
Cloud velocity, $\langle v^2 \rangle^{1/2}$	14 km sec ⁻¹
$\langle n_{\rm H} \rangle$	3 · 10 ⁻⁵ m ⁻³
Magnetic field	3 · 10 ⁻¹⁰ tesla

The gas consists mainly of hydrogen, about 10 percent ionized in H II clouds. The mean density in the galactic disk is almost constant up to $R \approx 2.47 \cdot 10^{20}$ m. It diminishes in the outer parts of the galaxy. Between the arms, $n_H \approx 7 \cdot 10^{-4}$ m⁻³. Due to a very small proportion of metal atoms, the gas is electrically conducting even in cold H I regions and able to freeze the magnetic field.

The interstellar cosmic ray density is approximately constant throughout the galaxy and equal to the value near the earth. The cosmic rays affect the galactic magnetic field in such a way as to generate a halo and to produce automatically cosmic ray isotropy in the galactic disk.

The interstellar magnetic field is frozen into the plasma and the cosmic ray gas. The three components are therefore coupled with each other. The motions within the medium are fluid-magnetic motions of the entire gas. In H I regions the damping of the motions is strong.

B. The Attenuation of X-Rays

Table 8 contains the attenuation coefficients for X-rays of a mixture of five elements between hydrogen and oxygen. For comparison, hydrogen, a hydrogen plus helium mixture, and all the ten elements used by Bell and Kingston [3] between hydrogen and argon may be seen in Table 1. They are all standardized to ten hydrogen atoms 10⁷ m⁻³. The elements contained in the mixtures in both tables are assumed to be present in the proportions used by Bell and Kingston.

Table 9 contains the corresponding transmittances of X-rays for an assumed distance which should be indicative of the extension of a standard cloud. It is seen that the hydrogen plus helium mixture would transmit all wavelengths between 0.01 to 20 · 10⁻¹⁰ m; but this mixture is unrealistic since no grains could be generated in it. The hydrogen to argon mixture would cut the transmission off at 2 · 10⁻¹⁰ m; this contradicts the experimental evidence. It is therefore concluded that the hydrogen to oxygen mixture should be applicable, and that the heavier elements from neon to argon are tied up not in H I clouds but in stellar plasmas and clouds. This mixture cuts the transmission off at $9 \cdot 10^{-10}$ m, which is lower than the values given in the literature but is provisionally acceptable. Doubtless this model of the attenuating medium will be varied again after more accurate experimental results have been obtained. According to this model, a cloud in space might consist of a hydrogen plus helium mixture in which are imbedded graphite particles surrounded with mantles of ices. The nitrogen is tied up in ammonia; 100 molecules of H₂ O are accompanied by 10 molecules of NH₃. If the graphite particles are assumed to be spheres, the radii are about 5 · 10⁻⁸ m. According to Wickramasinghe [7], with a mean graphite core radius of 5 · 10⁻⁸ m and a density of 2.2 · 10³ kg m⁻³ for graphite, a typical mantle can grow to at most 10^{-7} m with ice before most of the interstellar oxygen atoms are used up. Such a particle consists of 7.5 percent carbon and 92.5 percent water, if the ammonia is disregarded. Its total weight is $6.0 \cdot 10^{-17}$ kg.

C. Intergalactic Space

In intergalactic space, the density of grain material amounts to $\approx 10^{-28} \text{ kg m}^{-3}$. Compared with a mean density of grains near the galactic plane of $1.3 \cdot 10^{-23} \text{ kg m}^{-3}$, this is negligible. The attenuation in intergalactic space can therefore be disregarded.

TABLE 8. ATTENUATION COEFFICIENTS FOR X-RAYS OF A MIXTURE OF FIVE ELEMENTS BETWEEN HYDROGEN AND OXYGEN

Wavelength	Energy	Attenuation Coefficients (10 ² m ⁻¹)	
(10^{-10} m)	(keV)	Hydrogen to Oxygen	
		10 ⁻²³	
0.01	1239.85	0.2530	
0.02	619.927	0.3519	
0.03	413.285	0.4171	
0.03	309.964	0.4660	
0.05	247.971	0.5048	
0.03	247.571	0.50 10	
0.06	206.642	0.5367	
0.07	177.122	0.5635	
0.08	154.982	0.5864	
0.09	137.762	0.6062	
0.10	123.985	0.6236	
0.20	61.9927	0.7246	
0.30	41.3285	0.7717	
0.40	30.9964	0.8019	
0.50	24.7971	0.8272	
0.60	20.6642	0.8534	
0.70′	17.7122	0.8845	
0.80	15.4982	0.9239	
0.90	13.7762	0.9747	
1.0	12.3985	1.0402	
2.0	6.19927	3.3857	
3.0	4.13285	13.083	
4.0	3.09964	40.758	
5.0	2.47971	106.9	
6.0	2.06642	251.2	
7.0	1.77122	552.1	
/.0	1.17122	00 M · I	
8.0	1.54982	1171.4	
9.0	1.37762	2463.1	
10.0	1.23985	5269.3	
20.0	0.6199	2 499 812.	

TABLE 9. TRANSMITTANCE I/I_O OF X-RAYS FOR A DISTANCE $x=3.4818\cdot 10^{17}$ m IN A MEDIUM OF VARYING COSMIC COMPOSITION IN THE NARROW-BEAM APPROXIMATION

Wavelength (10 ⁻¹⁰ m)	Hydrogen plus Helium	Hydrogen to Oxygen	Hydrogen: to Argon
0.01	0.9999	0.9999	0.9999
0.02	0.9999	0.9999	0.9999
0.03	0.9999	0.9999	0.9999
0.04	0.9999	0.9999	0.9998
0.05	0.9998	0.9998	0.9998
0.06	0.9998	0.9998	0.9998
0.07	0.9998	0.9998	0.9998
0.08	0.9998	0.9998	0.9998
0.09	0.9998	0.9998	0.9998
0.10	0.9998	0.9998	0.9998
0.20	0.9998	0.9998	0.9998
0.30	0.9998	0.9997	0.9997
0.40	0.9997	0.9997	0.9997
0.50	0.9997	0.9997	0.9997
0.60	0.9997	0.9997	0.9997
0.70	0.9997	0.9997	0.9997
0.80	0.9997	0.9997	0.9996
0.90	0.9997	0.9997	0.9995
1.0	0.9997	0.9997	0.9994
2.0	0.9996	0.9989	0.9913
3.0	0.9993	0.9957	0
4.0	0.9985	0.9865	
5.0	0.9970	0.9646	
6.0	0.9945	0.9168	
7.0	0.9907	0.8171	
8.0	0.9852	0.6119	
9.0	0.9777	0.1839	
10.0	0.9677	0	
20.0	0.6643		

But another effect may become important for X-ray galaxies, the Doppler effect of expanding spaces. M. von Laue has proved the following theorem [8]:

If light waves which originate in the same initial state are compared in two similar spaces, in which one constantly retains the initial value R_0 of the radius of curvature R of the space, while in the other one R varies with time, then the luminosities in corresponding points at corresponding times vary inversely to the 4th power of the corresponding R-values.

The extragalactic X-ray source 3 C 273 is at a distance of 1.54 · 10²⁵ m. Therefore, from the accepted value of the Hubble constant, it follows that:

$$\frac{1}{R}$$
 $\frac{dR}{dt}$ = 3.2 · 10⁻¹⁸ sec⁻¹

$$\frac{\Delta R}{R} = 3.2 \cdot 10^{-18} \ \Delta t = 3.2 \cdot 5.15 \cdot 10^{-18+16} = 1.65 \cdot 10^{-1}$$

It is seen that for 3 C 273, R increases during the passage of the trajectory of light by about 16 percent.

$$(1.16)^4 \approx 1.81$$

The luminosity of the nebula decreases, therefore, by the increase of $\,R\,$ by about 11/20.

The effect is not important if R increases by less than 10 percent.

III. THE PHYSICAL STATE OF AN H I CLOUD

A. Introduction

Between the stars exists the interstellar medium consisting of gas, dust, cosmic rays, and magnetic fields. It can be considered as a composite fluid. Its composition and structure are of decisive importance for the attenuation of X-rays. Other factors which influence the structure are temperature and electric fields.

B. The Interstellar Medium

The assumption is made that the interstellar space contains matter which is a mixture of the light elements, with relative abundances according to the cosmic distribution of elements. The values which will be adopted are contained in Table 10 [9].

TABLE 10. RELATIVE ABUNDANCES OF SOME OF THE LIGHT ELEMENTS

Atomic Number	Chemical Symbol	Relative Number of Atoms
1	Н	1
2	He	1.2 · 10 ⁻¹
6	C	2.5 · 10 ⁻⁴
7	N	2.5 · 10 ⁻⁴ 1.3 · 10 ⁻⁴
8	0	7.9 · 10-4

Some of the elements condense into dust. Cosmic rays and magnetic fields are also present. The gas can be considered to be pure hydrogen and helium. The other components form the dust particles. One atom of metals in 2000 is present, which keeps the gas ionized. In the galactic plane within a radius of $3.09 \cdot 10^{20}$ m, the average atomic concentration of interstellar hydrogen is approximately 7.10^5 m⁻³, the density inside a cloud is about 10^7 m⁻³, and the density of the intercloud medium about 10^5 m⁻³.

Cosmic dust consists of small solid particles with a radius of the order of 10^{-7} m. The particles possess a core of graphite with a radius of the order of $5 \cdot 10^{-8}$ m on which are adsorbed H_2 O and NH_3 .

The mean value of the magnetic field strength is approximately $3 \cdot 10^{-10}$ tesla.

The energy densities of the interstellar radiation, of the interstellar magnetic fields, of cosmic rays, and of kinetic energy of the interstellar gas are quantities of the same order of magnitude. This provides evidence that a close correlation exists between all the components of the interstellar medium. The mean energy density of the primary cosmic rays is approximately $5 \cdot 10^{-14}$ J m⁻³.

The interstellar medium cannot be considered to be a completely ionized gas.

C. The H I Clouds

THe interstellar gas is not evenly distributed but is present as clouds. The average size of a cloud is of the order of $3.09 \cdot 10^{17}$ m. A line of sight crosses usually about ten clouds in $3.09 \cdot 10^{19}$ m. Interstellar clouds disperse and are reformed. Because of the chemisorbent properties of a graphite surface, all interstellar atoms except helium will probably be chemisorbed and stick on the surface.

But since an H I cloud is involved in a cloud-cloud collision once every $(3.156 \cdot 10^7)^7$ sec and is, on the average, transformed into an H II cloud once every $(3.156 \cdot 10^7)^8$ sec, all graphite core-ice mantle grains present in an H I cloud are reduced to the graphite cores when the cloud is transformed into an H II region. The graphite cores themselves remain intact.

If a density of $2.25 \cdot 10^3$ kg m⁻³ is accepted for graphite, a sphere of radius $5 \cdot 10^{-8}$ m will have a mass of $1.18 \cdot 10^{-8}$ kg. According to Table 10, this implies a grain density of $N_{gr} = 4.23 \cdot 10^{-5}$ m⁻³. Further, if all the available oxygen and nitrogen are adsorbed, and if a density of $0.65 \cdot 10^3$ kg m⁻³ is accepted for ammonia, it implies masses of $5.58 \cdot 10^{-18}$ kg H_2 O and $8.69 \cdot 10^{-19}$ kg NH₃. With these amounts of water and ammonia, the radius becomes $1.21 \cdot 10^{-7}$ m.

D. The Stability of H I Clouds

The hypothesis is made that the galactic field in the disk of the galaxy is contained by the weight of the gas throughout the disk [10]. The average field in the disk is parallel to the plane of the disk. If a large-scale field along the galactic disk or arm and the cosmic rays are confined by the weight of the gas, then the gas tends to drain downward along the magnetic lines of force into the lowest region, thereby releasing the field between the low regions to expand upward. The cloud masses inferred from the observations are not large enough to maintain their equilibrium by self-gravitation, and the magnetic field and the cosmic ray gas drive the interstellar gas-field system unstable in periods of about $(3.156 \cdot 10^7)^7$ sec. The interstellar gas is suspended in the field in discrete clouds with separations of the order of 10 to $10^3 \cdot 3.0856$ m. As a result of the instabilities, turbulence develops.

In a plasma confined by a magnetic field whose value remains constant in time at each point, the diffusion velocity across the magnetic field, resulting from collisions of electrons with ions, can be obtained. The transverse diffusion velocity resulting from finite resistivity η is given by Spitzer [11] in the form

$$\nu_{\eta} = -\frac{\eta}{R^2} \quad \forall p \tag{4}$$

The outward motion of a confined plasma across a magnetic field is referred to as "collisional diffusion." Encounters between identical particles do not produce any appreciable diffusion.

In the presence of turbulent fields, the diffusion of plasma across the field lines can be increased above the collisional diffusion rate given by equation (4). If this process is unrelated to collisions between particles, the magnitude of the diffusion velocity is computed from the following dimensional argument [11]:

If the particle flux is given by the product of a diffusion coefficient D and the density gradient ∇n , then

$$\overrightarrow{n\nu} = \mathbf{D} \nabla \mathbf{n}$$

where D has the dimensions of velocity times distance. In diffusion of neutral atoms through a gas

$$D \approx \lambda \omega$$

where λ denotes the mean free path and ω the thermal velocity. The diffusion velocity across a magnetic field, given in equation (4), corresponds to

$$D = \lambda \omega \left(\frac{a}{\lambda}\right)^2$$

where a denotes the radius of gyration. The simplest form for $\,D\,$ which is not related to $\,\lambda\,$ is

$$D = a \omega$$

The radius of gyration is given by

$$a = \frac{m\omega_{\perp}}{eB}$$

Substituting this value into the preceding equation yields

$$D = \frac{kT}{eB}$$

The result is divided by 16; this divisor is considered as uncertain by Spitzer, since dimensional analysis does not yield it and no other derivation is known. Denoting the diffusion velocity resulting from plasma turbulence by ν_{t} , the final result is:

$$v_{\rm t} = -\frac{kT}{16e \, n_{\rm e} B} \quad \nabla n_{\rm e} \tag{5}$$

The following values were used in order to evaluate equation (4)

$$\eta = 5.16 \Omega m$$
 (from Spitzer [11], equation 5-42)

$$\ln \Lambda = 40$$
 (from Kaplan and Pikelner [12])

$$T = 100^{\circ}K$$

$$B = 3 \cdot 10^{-10} \text{ tesla}$$

$$p = \frac{B^2}{2 \mu_0} = 3.581 \cdot 10^{-14} \,\text{N/m}^2$$

$$\ell = 10 \text{ pc} = 3.0856 \cdot 10^{17} \text{ m}$$
 (characteristic distance)

 ∇p was approximated by $\frac{p}{\varrho}$.

In equation (5), ∇n_e was approximated by $\frac{n_e}{\ell}$. Then the factor n_e cancels and the other plasma parameters are the same as for equation (4). The following values were obtained:

$$\nu_{\eta} = 6.65 \cdot 10^{-12} \text{ m sec}^{-1}$$

$$\nu_{t} = 5.82 \cdot 10^{-12} \text{ m sec}^{-1}$$

It is seen that under the assumptions made the turbulent diffusion rate is of the same order of magnitude as the collisional diffusion rate.

REFERENCES

- 1. Grodstein, G. W.: X-Ray Attenuation Coefficients from 10 keV to 100 MeV. NBS Circular, no. 583, 1957.
- 2. Aller, L. H.: The Abundance of the Elements. Interscience Publishers (New York-London), 1961.
- 3. Bell, K. L. and Kingston, A. E.: The Absorption of X-Rays by Interstellar Gas. Mon. Not. R. Ast. Soc., vol. 136, 1967, pp. 241-244.
- 4. Landolt-Boernstein, Astronomy and Astrophysics. H. H. Voigt, ed., (Berlin-Heidelberg-New York), VI/I, 1965.
- 5. Spitzer, L.: Diffuse Matter in Space. Interscience Publishers (New York-London-Sydney-Toronto), 1968.
- 6. Hottel, H. C., Sarofim, A. F., and Sze, D. K.: Radiation Transfer in an Absorbing-Scattering Medium. Proc. 3rd Int. Heat Transf. Conf. (AIChE, New York), vol. 5, 1966, pp. 112-121.
- 7. Wikramasinghe, N. C.: Interstellar Grains. Chapman and Hall (London), 1967.
- 8. von Laue, Max: The Propagation of Light in Spaces with Temporally Variable Curvature According to the General Theory of Relativity. Sitzungsber. d. Preuss. Akad. d. Wiss. Math. Phys. Kl., 1931, pp. 123-131.
- Greenberg, J. Mayo: Interstellar Grains, Nebulae and Interstellar Matter.
 B. M. Middlehurst and L. H. Aller, ed., University of Chicago Press, 1968, pp. 221-364.
- 10. Parker, E. N.: The Dynamical State of the Interstellar Gas and Field. Astrophys. J., vol. 145, 1966, pp. 811-833
- 11. Spitzer, L.: Physics of Fully Ionized Gases. Interscience Publishers (New York, London), 1962.
- 12. Kaplan, S. A. and Pikelner, S. B.: The Interstellar Medium. Harvard University Press (Cambridge), 1970.

APPROVAL

THREE INVESTIGATIONS OF THE INTERSTELLAR MEDIUM

By Klaus Schocken

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

WILLIAM C. SNODDY

Chief, Space Thermodynamics Division

ÆRHARD B. HELLER

Director, Space Sciences Laboratory

DISTRIBUTION

INTERNAL

DIR

AD-S

Dr. Ernst Stuhlinger

S&E-DIR

Mr. Hermann Weidner

S&E-R

Dr. William Johnson

PD-MP

Mr. James Downey

S&E-AERO-T

Dr. Helmut Krause

S&E-SSL-DIR

Mr. Gerhard Heller Mr. Ray Hembree

S&E-SSL-C

Mr. James Mathis Reserve (15)

S&E-SSL-N

Mr. Henry Stern

S&E-SSL-P

Dr. Robert Naumann

S&E-SSL-S

Dr. Werner Sieber

S&E-SSL-T

Mr. William Snoddy

Dr. Klaus Schocken (10)

S&E-SSL-TE

Mr. Ed Miller

Mr. Harry Atkins

Mr. Walter Fountain

Mr. James Fountain

Mr. Stanley Fields

Mr. John Reynolds

Mr. Charles Baugher

Mr. Ed Reichmann

Mr. Robert Wilson

S&E-SSL-TR

Mr. Gary Arnett

Mr. Tommy Bannister

Dr. Roger Kroes

Mr. Roger Linton

Mr. Donald Wilkes

Mr. James Zwiener

Miss Barbara Richard

Dr. Ulrich Wegner

Dr. Kulshreshtha

S&E-SSL-TT

Mr. Billy Jones

Mr. Daniel Gates

Mr. Paul Craven

Mr. Jimmy Watkins

Dr. Mona J. Hagyard

Mr. Ted Calvert

Dr. Gilmer Gary

S&E-SSL-X

Dr. Rudolf Decher

Mr. Hoyt Weathers

DEP-T

A&TS-PAT

Mr. L. D. Wofford, Jr.

DISTRIBUTION (Concluded)

INTERNAL (Concluded)

PM-PR-M A&TS-MS-IP (2) A&TS-MS-IL (8) A&TS-TU (6) A&TS-MS-H

EXTERNAL

National Aeronautics and Space Administration Washington, D. C. 20546

Attn: Dr. A. Boggess, Code 613 Dr. K. Hallan, Code 613 Dr. T. Stecher, Code 613 Dr. J. Underwood, Code 614 Dr. E. Boldt, Code 611

Manned Spacecraft Center Houston, Texas 77058 Attn: Dr. Yoji Kondo, TG4

Scientific and Technical Information Facility (25) P. O. Box 33
College Park, Maryland 20740
Attn: NASA Representative (S-AK/RKT)

Dr. Edward J. Devinney Department of Astronomy University of South Florida Tampa, Florida 33620

Dr. Laurence E. Peterson Department of Physics Space Physics Group University of California La Jolla, California 92037

26